

Fast Polyhedra Abstract Domain



Gagandeep Singh



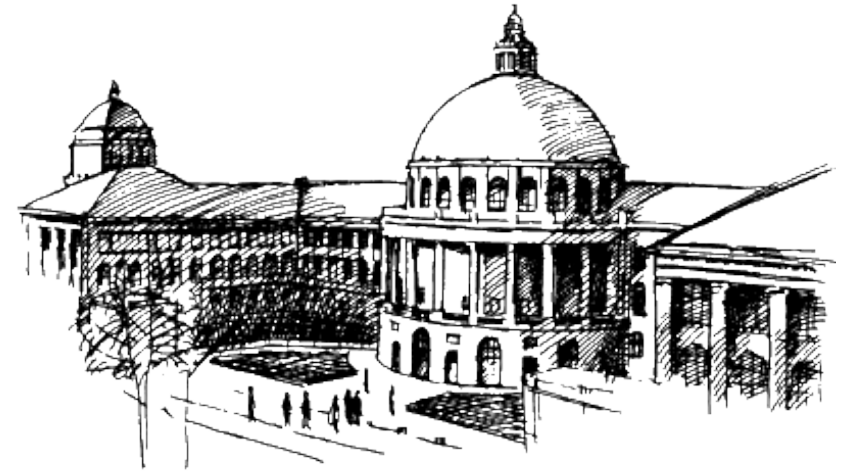
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Polyhedra Domain Analysis

Automatic Discovery of Linear Restraints Among Variables of a Program, POPL'78

Polyhedra Domain Analysis

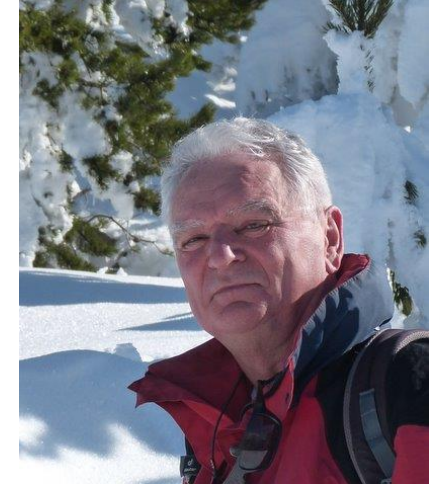
Automatic Discovery of Linear Restraints Among Variables of a Program, POPL'78

Introduced by Patrick Cousot and
Nicolas Halbwachs

Represents linear constraints
between program variables



Patrick Cousot



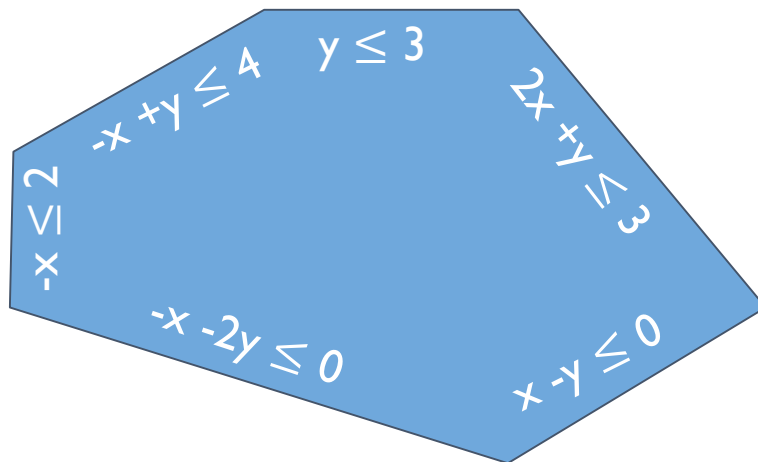
Nicolas Halbwachs

Polyhedra Domain Analysis

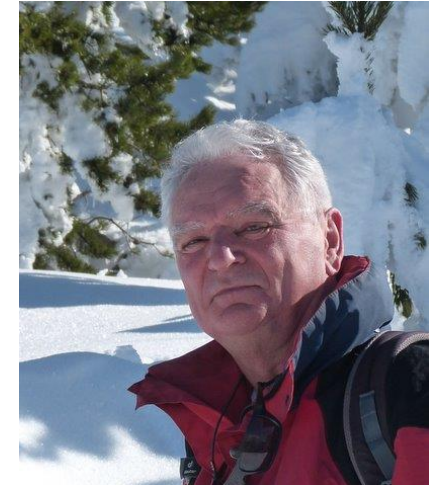
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Polyhedra Domain Analysis

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if(*){  
    y:=2x-1;  
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Abstract Domain	Can Prove the Assertion?
Interval	✗
Pentagon	✗
Zones	✗
Octagon	✗
Polyhedra	✓

Polyhedra Domain Analysis

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Polyhedra analysis: time and space exponential in number of variables

This work: contributions

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Online decomposition:

reduction in space and time
without losing precision

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Constant factor improvements
via reduced operation count and
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ELNA elina.ethz.ch

Complete end-to-end
implementation

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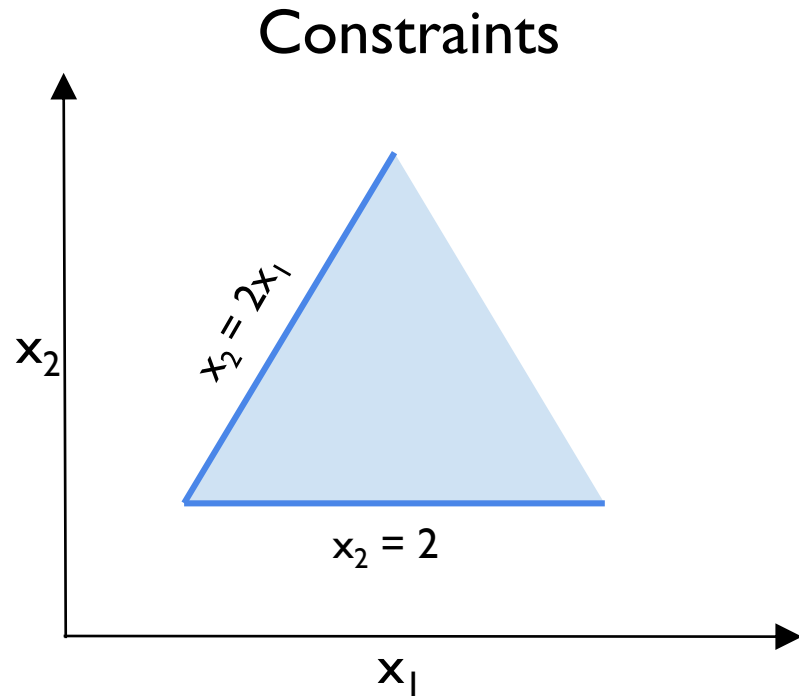
Complete end-to-end
implementation

Constant factor improvements
via reduced operation count and
cache optimizations

Driver	NewPolka	PPL	ELINA
➤ 500 var ➤ 39K LOC	OOM (> 12 GB)	OOM (> 12 GB)	4 sec 0.9 GB
➤ 650 var ➤ 25K LOC	TO (> 4 hr)	TO (> 4 hr)	2 sec 0.4 GB

Double Representation of Polyhedron

Double Representation of Polyhedron

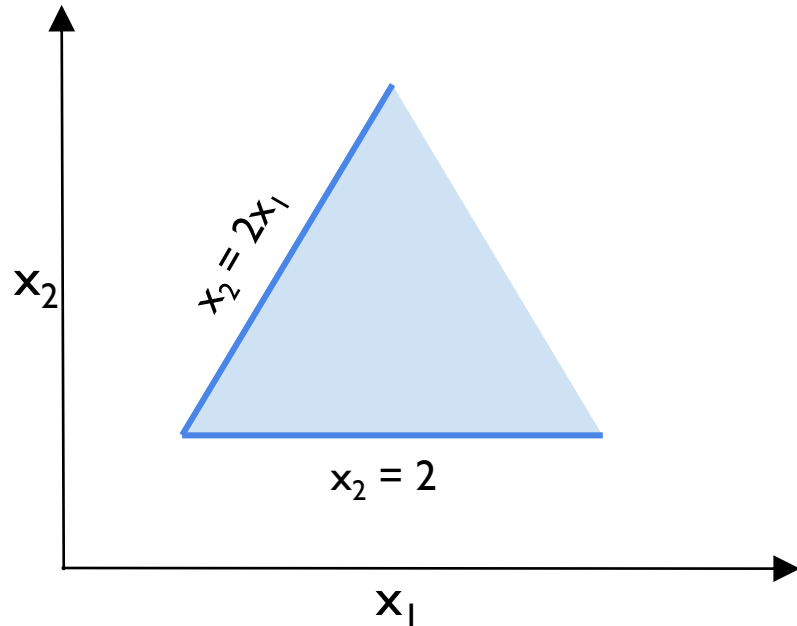


$$\mathcal{C} = \{-x_2 \leq -2, x_2 \leq 2x_1\}$$

m: number of constraints

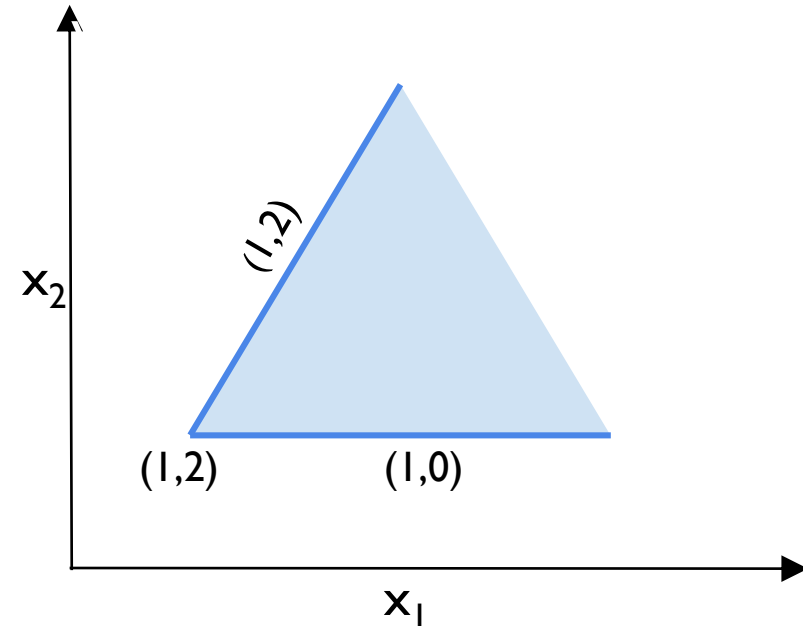
Double Representation of Polyhedron

Constraints



$\mathcal{C} = \{-x_2 \leq -2, x_2 \leq 2x_1\}$
m: number of constraints

Generators



Vertices $\mathcal{V} = \{(1,2)\}$,
Rays $\mathcal{R} = \{(1,2), (1,0)\}$,
Lines $\mathcal{Z} = \emptyset$
g: number of generators

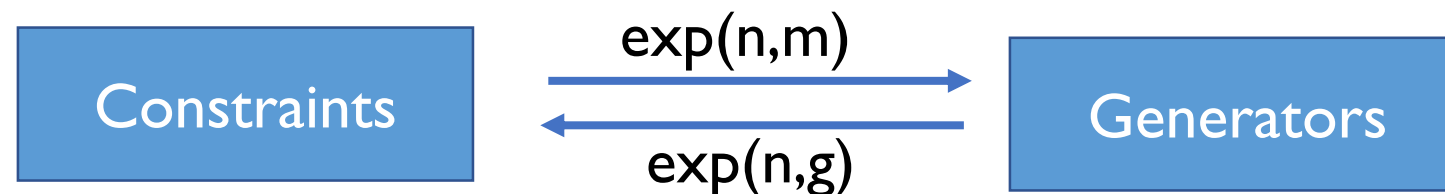
Asymptotic Time Complexity of Polyhedra

Asymptotic Time Complexity of Polyhedra

Operator	Constraints	Generators	Both
Join (\sqcup)	$\exp(n,m)$	$O(n g)$	$O(n g)$
Meet (\sqcap)	$O(n m)$	$\exp(n,g)$	$O(n m)$
Inclusion (\sqsubseteq)	$\exp(n,m)$	$\exp(n,g)$	$O(n g m)$
Assignment	$O(n m^2)$	$O(n g)$	$O(n g)$
Conditional	$O(n)$	$\exp(n,g)$	$O(n)$

Asymptotic Time Complexity of Polyhedra

Operator	Constraints	Generators	Both
Join (\sqcup)	$\exp(n,m)$	$O(n,g)$	$O(n,g)$
Meet (\sqcap)	$O(nm)$	$\exp(n,g)$	$O(nm)$
Inclusion (\sqsubseteq)	$\exp(n,m)$	$\exp(n,g)$	$O(n,g,m)$
Assignment	$O(nm^2)$	$O(n,g)$	$O(n,g)$
Conditional	$O(n)$	$\exp(n,g)$	$O(n)$



Key Idea: Online Decomposition

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Polyhedron

$$\{x_1 \leq 2x_2,$$
$$x_2 = 2,$$
$$x_1 + x_2 + 2x_3 \leq 5,$$

$$x_4 - x_5 \leq 3,$$
$$x_5 = 1,$$

$$x_6 = 2\}$$

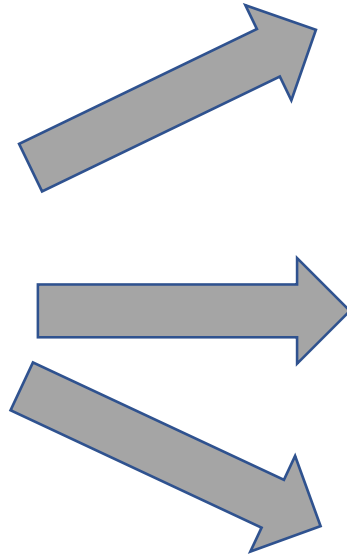
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Set of factors

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Key Idea: Online Decomposition

Partition (π) =
set of blocks

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$$\{x_1, x_2, x_3\}$$

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Key Idea: Online Decomposition

Partition (π) =
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Polyhedron

$$\begin{cases} x_1 \leq 2x_2, \\ x_2 = 2, \\ x_1 + x_2 + 2x_3 \leq 5, \\ x_4 - x_5 \leq 3, \\ x_5 = 1, \\ x_6 = 2 \end{cases}$$

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working on smaller Polyhedra enables reduction in space and time

Permissible Partitions

Permissible Partitions

Polyhedron

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Permissible Partitions

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Best (finest)
partition (π)

$$\{x_1, x_2, x_3\}$$

$$\{x_4, x_5\}$$

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Permissible Partitions

Polyhedron

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Permissible
partition ($\bar{\pi}$)

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$$\{x_4, x_5, x_6\}$$

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$$\{x_4, x_5, x_6\}$$

Invalid
partition

$$\{x_1, x_2\}$$

$$\{x_3, x_4, x_5\}$$

$$\{x_6\}$$

Permissible Partitions

Polyhedron	Best (finest) partition (π)	Permissible partition ($\bar{\pi}$)	Invalid partition
$\{x_1 \leq 2x_2,$ $x_2 = 2,$ $x_1 + x_2 + 2x_3 \leq 5,$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2, x_3\}$	$\{x_1, x_2\}$
$x_4 - x_5 \leq 3,$ $x_5 = 1,$	$\{x_4, x_5\}$	$\{x_4, x_5, x_6\}$	$\{x_3, x_4, x_5\}$
$x_6 = 2\}$	$\{x_6\}$		$\{x_6\}$

Definition: A partition π is *permissible* for Polyhedron P , if there are no two variables x_i and x_j in different blocks of π related by a constraint in P

Partition of Variable Set: Summary

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The set of all partitions of variable set \mathcal{X} form a lattice ordered by “*finer than*” ($<$) relation

The **best (finest)** partition π_P for Polyhedron P is unique

Any $\bar{\pi}$, s.t., $\pi_P < \bar{\pi}$, is permissible

An unconstrained variable x_i yields a singleton set $\{x_i\}$ in the partition

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Challenge: maintain permissible partitions for > 30 operators

Operator: Conditional

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Definition: Let π be a partition and \mathcal{B} be a block, then $\pi \uparrow \mathcal{B}$ is the finest partition π' such that $\pi \sqsubseteq \pi'$ and \mathcal{B} is a subset of an element of π'

Theorem (finest partition after conditional):

If $O \neq \perp$ and let \mathcal{B} be block containing all variables appearing in the conditional, then $\pi_O = \pi_P \uparrow \mathcal{B}$

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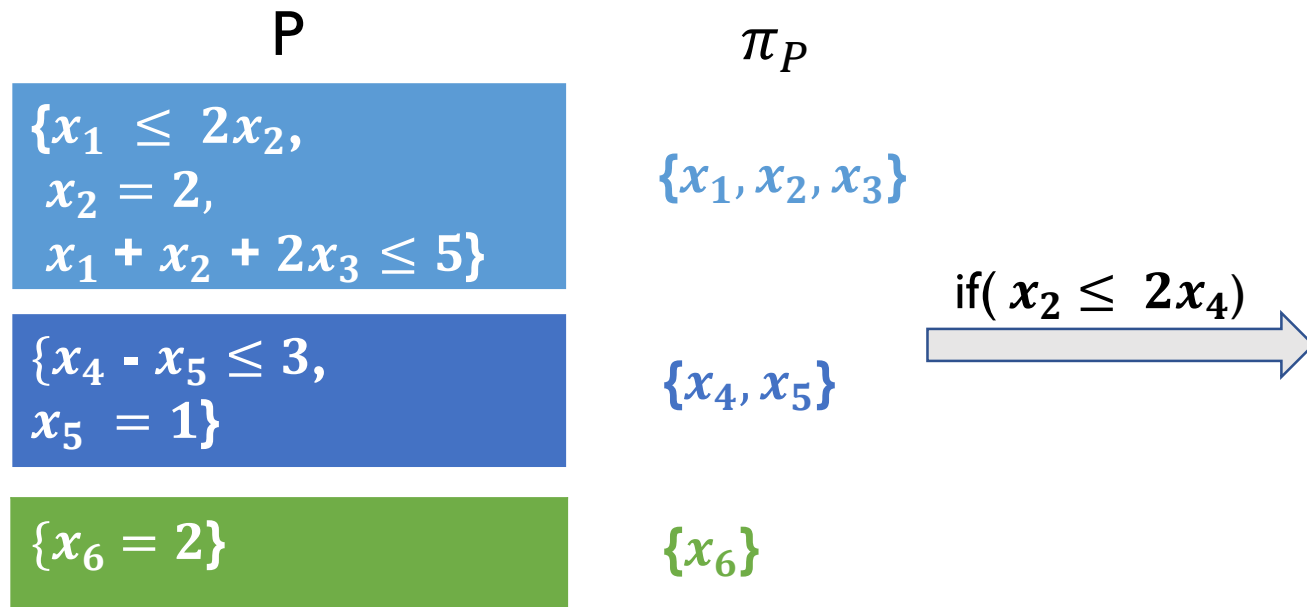
\mathcal{P}	$\pi_{\mathcal{P}}$
$\{x_1 \leq 2x_2,$ $x_2 = 2,$ $x_1 + x_2 + 2x_3 \leq 5\}$	$\{x_1, x_2, x_3\}$
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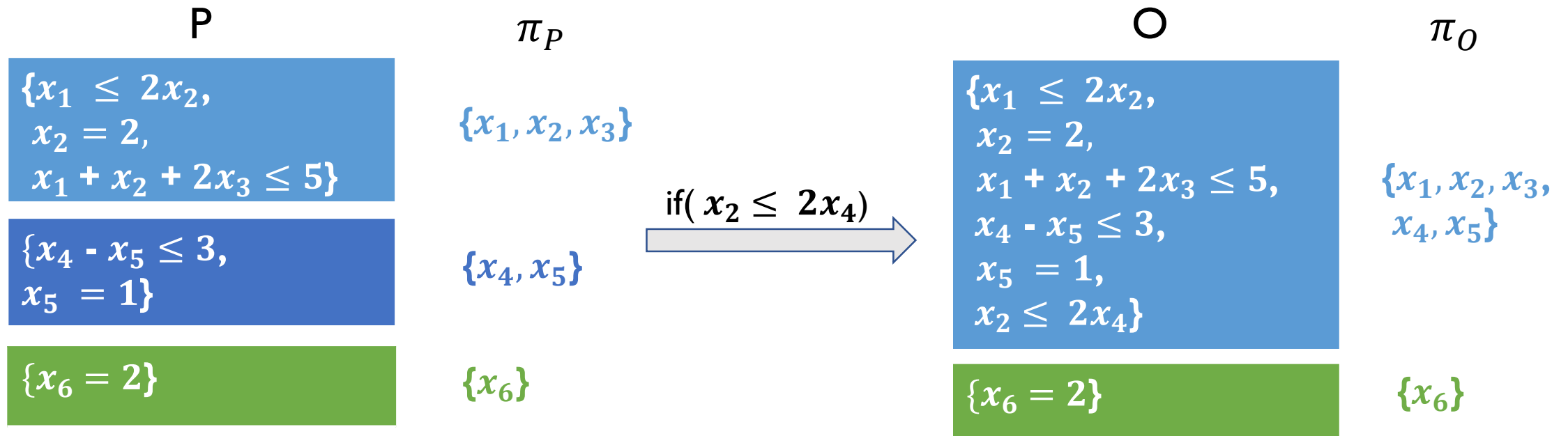


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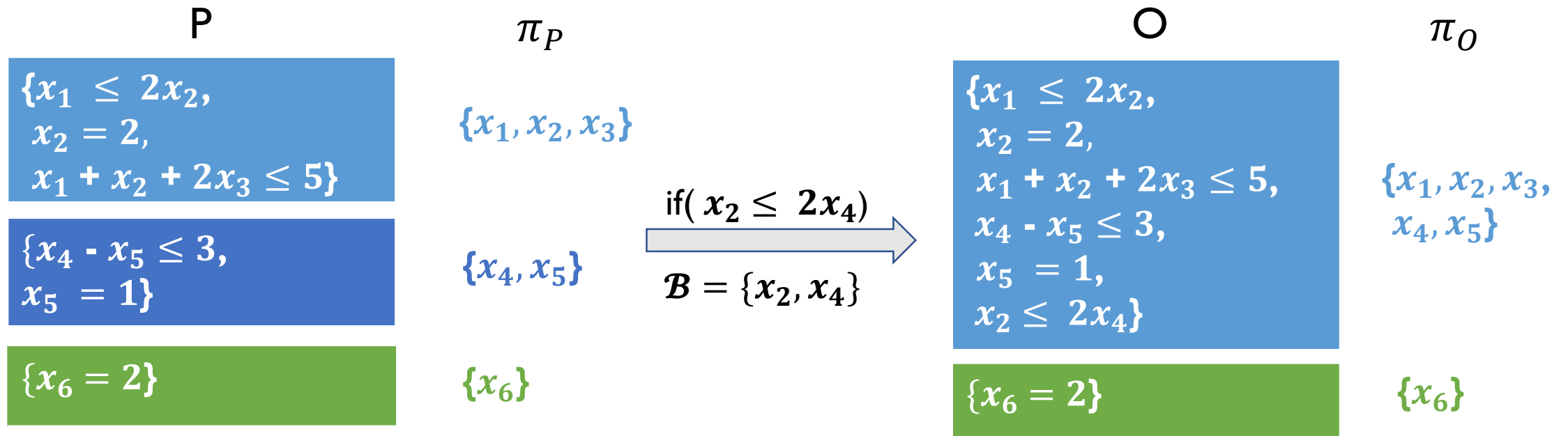


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Operator: Assignment

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P

π_P

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$$\{x_1, x_2, x_3\}$$

$$\{x_4 - x_5 \leq 3, \\ x_5 = 1\}$$

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Operator: Assignment

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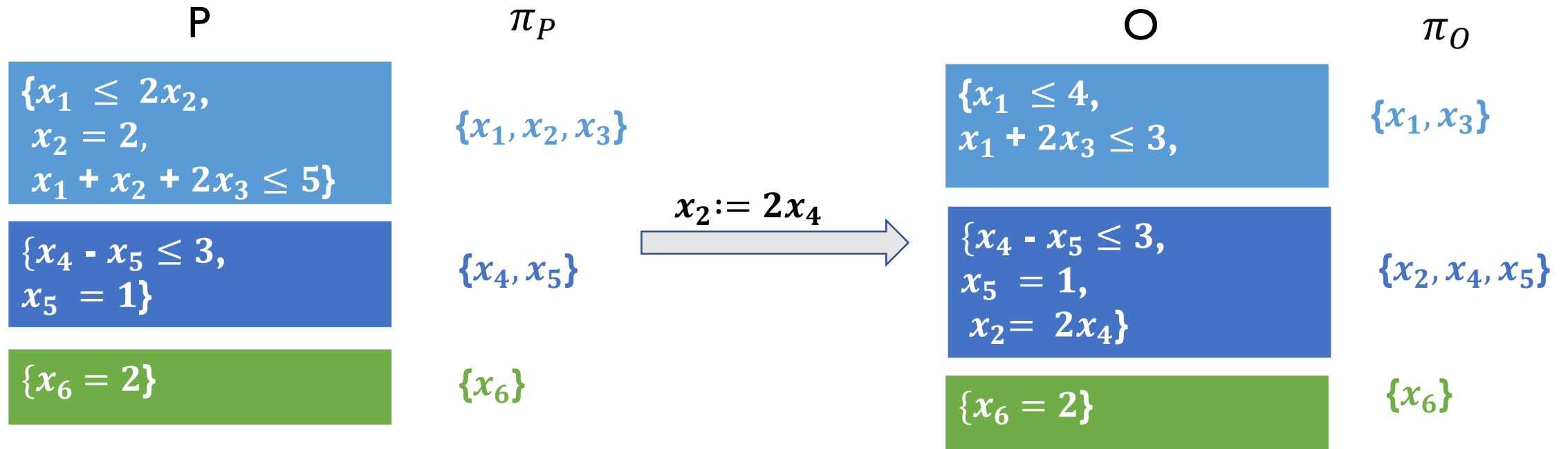
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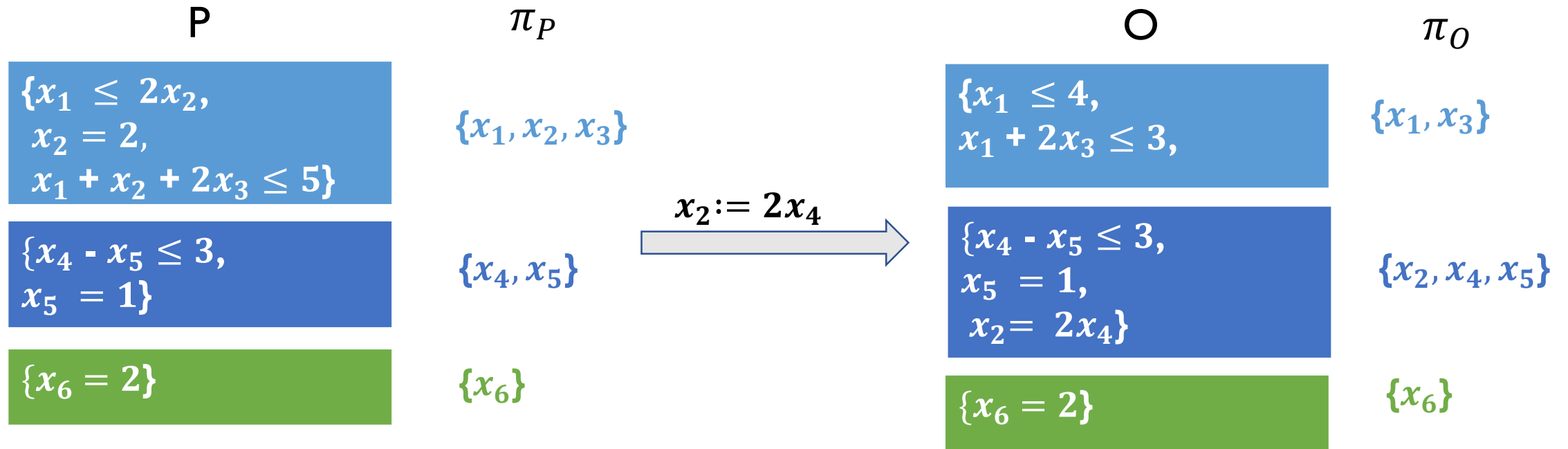
$$x_2 := 2x_4$$



Operator: Assignment



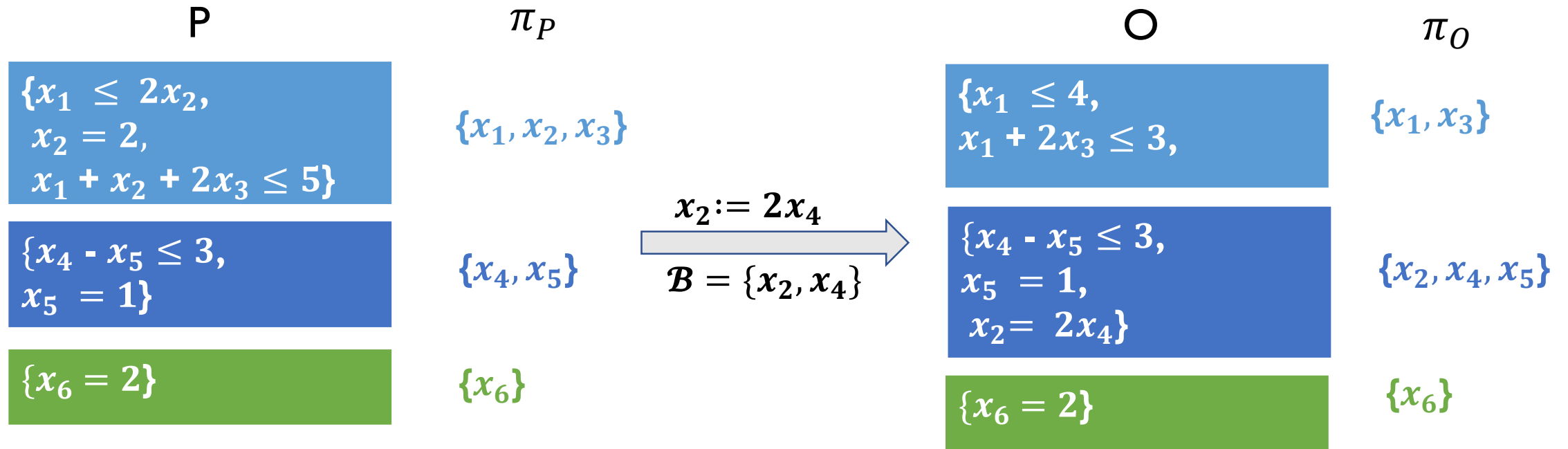
Operator: Assignment



Theorem (finest partition after assignment):

Let \mathcal{B} be block containing all variables appearing for assignment $x_i := e$, and let $\pi_i = \{\mathcal{X} \setminus \{x_i\}, \{x_i\}\}$, then $\pi_O = (\pi_P \sqcap \pi_i) \uparrow \mathcal{B}$

Operator: Assignment



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Lattice Operators

Lattice Operators

Theorem (finest partition for \sqsubseteq):

If $P \sqsubseteq Q$ and $P \neq \perp$, then $\pi_Q \sqsubseteq \pi_P$

Theorem: (finest partition after \sqcap):

If $P \sqcap Q \neq \perp$, then $\pi_O = \pi_P \sqcup \pi_Q$

For join (\sqcup), no general relationship exists between π_O , π_P and π_Q

Operator: Join (\sqcup)

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P

$\{x_1 - x_2 \leq 0,$
 $x_1 \leq 0\}$

π_P

$\{x_1, x_2\}$

$\{x_3 = 1\}$

$\{x_3\}$

Operator: Join (\sqcup)

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π_P

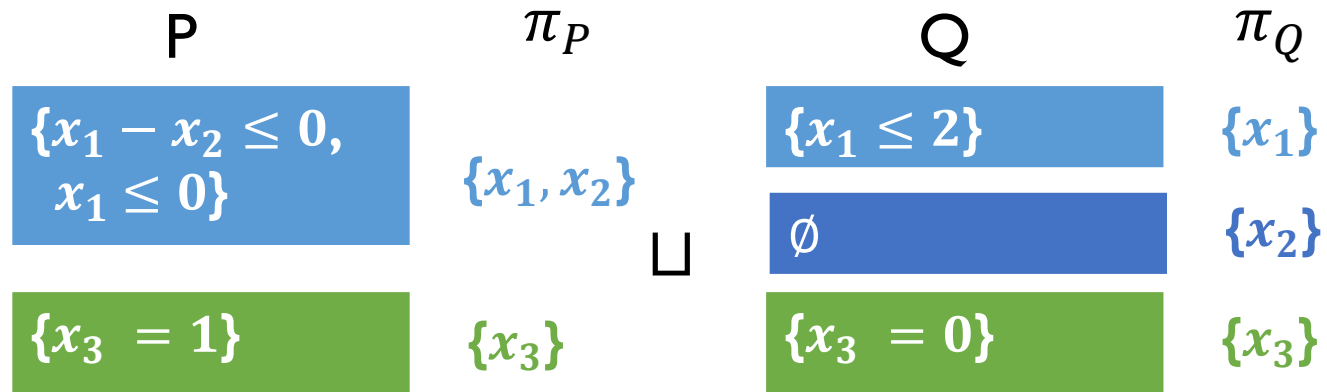
$\{x_1, x_2\}$

\sqcup

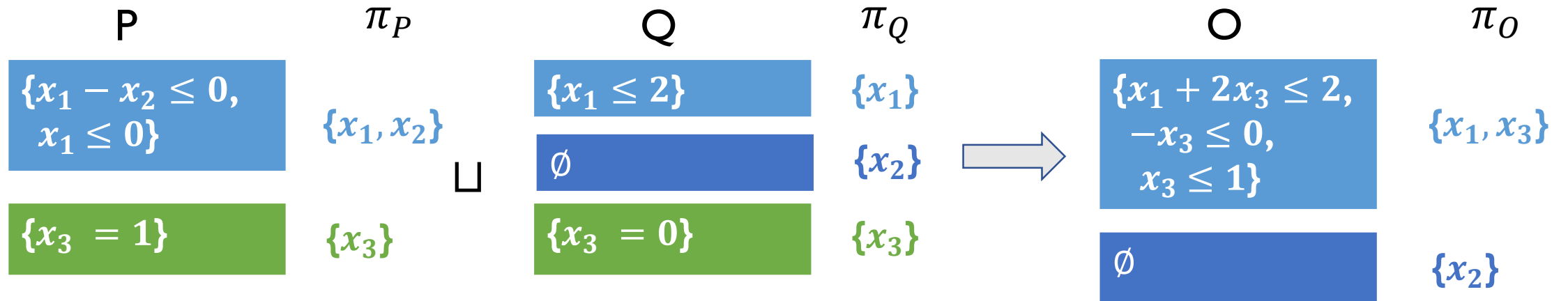
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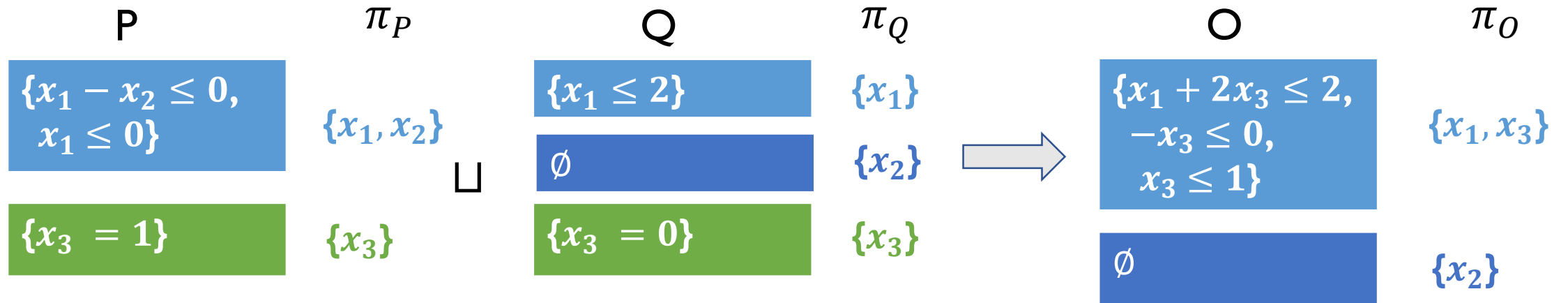
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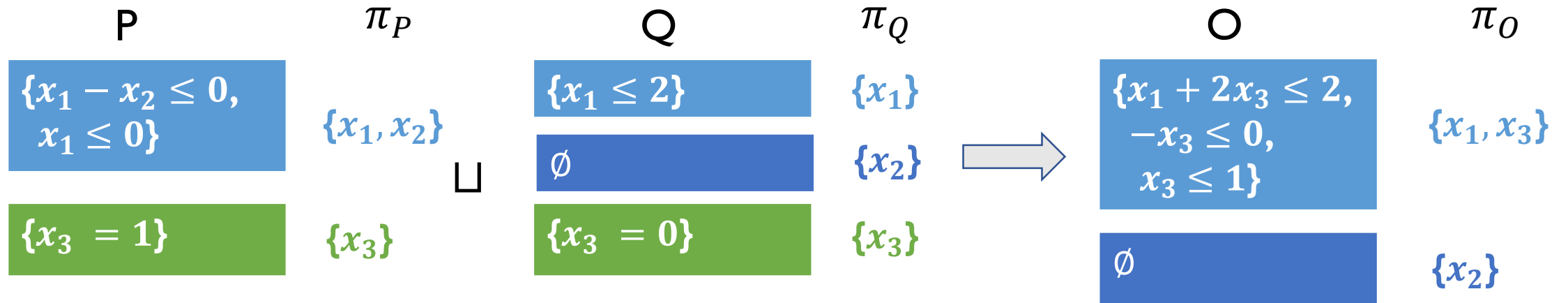


Operator: Join (\sqcup)



$$\pi_P \sqcup \pi_Q = \pi_P \neq \pi_O$$

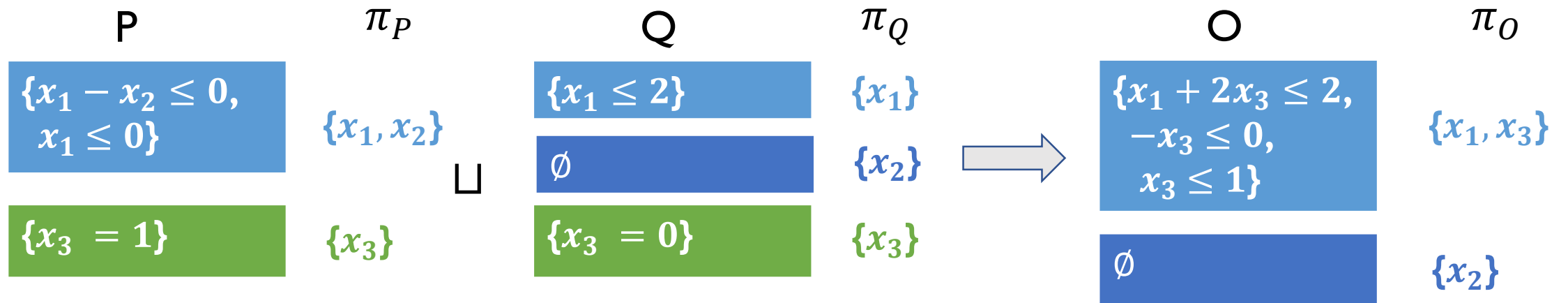
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$$\pi_P \sqcap \pi_Q = \pi_Q \neq \pi_O$$

Operator: Join (\sqcup)



$$\pi_P \sqcup \pi_Q = \pi_P \neq \pi_O$$

$$\pi_P \sqcap \pi_Q = \pi_Q \neq \pi_O$$

For Join, π_O depends on both P and Q

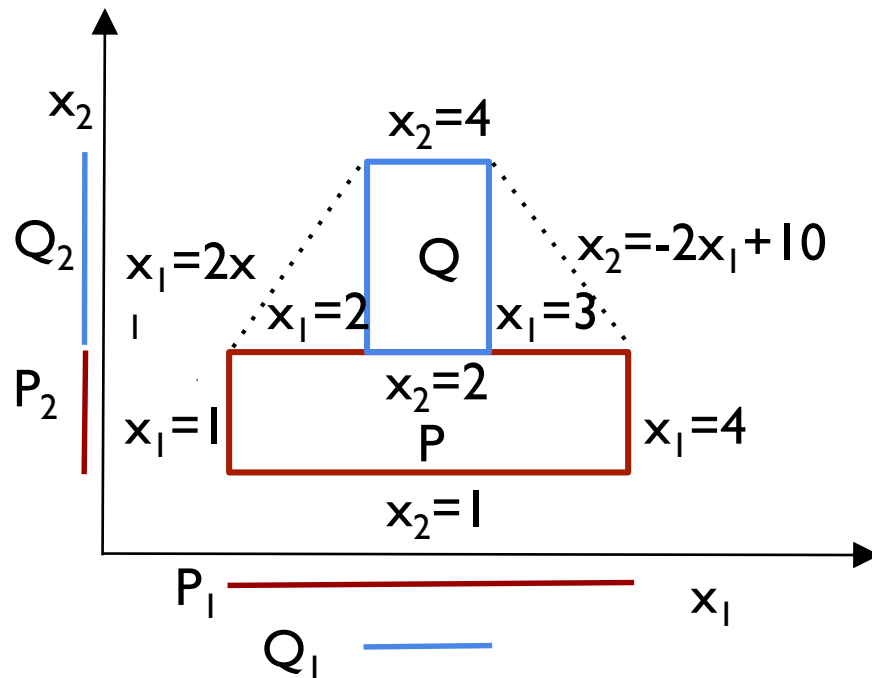
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Theorem: Let P and Q be two Polyhedra with the same permissible partition $\pi = \{X_1, X_2, \dots, X_r\}$ and let $\bar{\pi}$ be a permissible partition for the join, that is, $\pi_{P \sqcup Q} \sqsubseteq \bar{\pi}$. If for any block $X_k \in \pi$, $P_k = Q_k$, then $X_k \in \bar{\pi}$

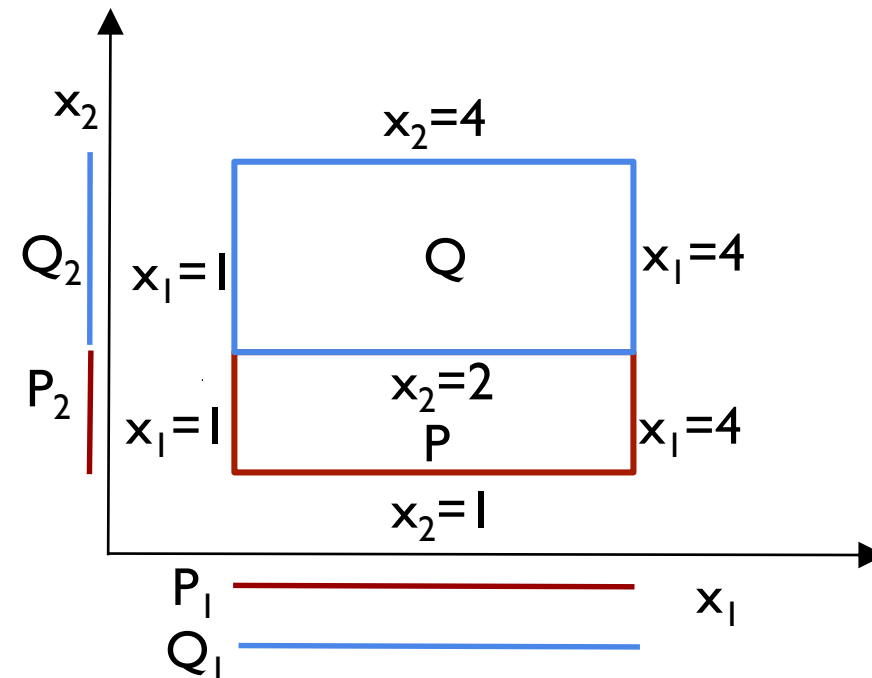
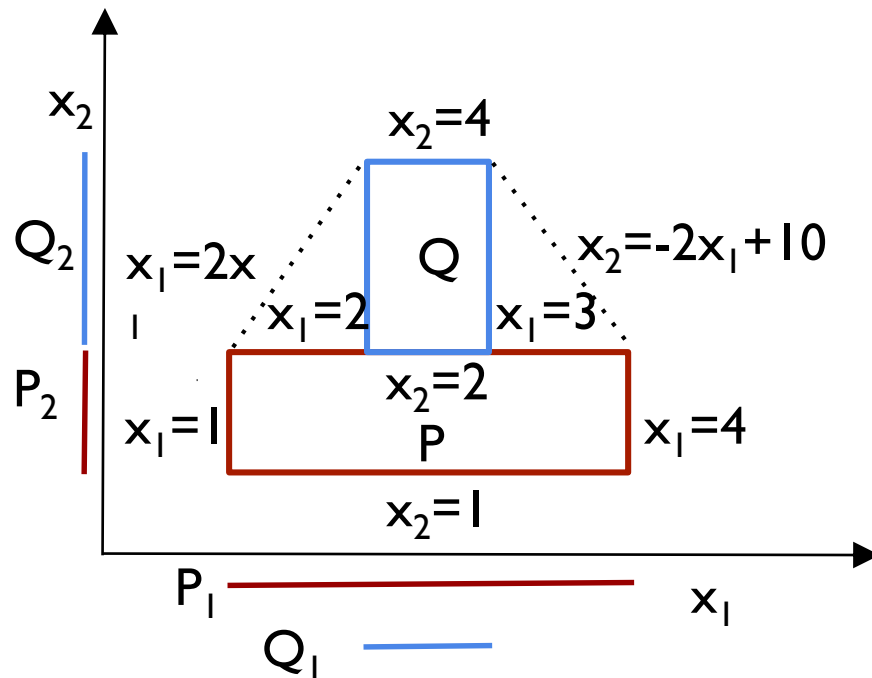
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Operators with Permissible Partitions

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Theorem (permissible partition after join):

Let $\bar{\pi} = \bar{\pi}_P \sqcup \bar{\pi}_Q$ and $\mathcal{U} = \{X_k \mid P_k = Q_k, X_k \in \bar{\pi}\}$.

Then $\bar{\pi}_{P \sqcup Q} = \mathcal{U} \cup \bigcup_{\mathcal{J} \in \bar{\pi} \setminus \mathcal{U}} \mathcal{J}$ is permissible for $P \sqcup Q$

Theorem (permissible partition after meet):

$\bar{\pi}_P \sqcup \bar{\pi}_Q$ is permissible for $P \sqcap Q$

Theorem (permissible partition after conditional):

If output $O \neq \perp$, then, $\bar{\pi}_P \uparrow \mathcal{B}$ is permissible for conditional

Theorem (permissible partition after assignment):

$\bar{\pi}_P \uparrow \mathcal{B}$ is permissible for the output O of assignment

Asymptotic Complexity of Operators with Permissible Partitions

Operator	Before (using both)	Our work (using decomposition)
Join (\sqcup)	$O(n g)$	$O(\sum_{i=1}^r n_i m_i g_i + n_{max} m_{max})$
Meet (\sqcap)	$O(n m)$	$O(\sum_{i=1}^r n_i m_i)$
Inclusion (\sqsubseteq)	$O(n g m)$	$O(\sum_{i=1}^r n_i m_i g_i)$
Assignment	$O(n g)$	$O(n_{max} g_{max})$
Conditional	$O(n)$	$O(n_{max})$
Conversion	$exp(n, g)$	$exp(n_{max}, g_{max})$

r : number of blocks

Experimental Evaluation

We compared performance of ELINA against NewPolka and PPL

Using the Seahorn verification framework [CAV'15]

- written in C, analyzes llvm-bitcode
- produces Polyhedra invariants

> 1500 benchmarks from the software verification competition

Time limit: 4 hours

Memory limit: 12 GB

Experimental Evaluation

Experimental Evaluation

Benchmark	Category	LOC	NewPolka		PPL		ELINA		Speedup ELINA vs.	
			time(s)	memory(GB)	time(s)	memory(GB)	time(s)	memory(GB)	NewPolka	PPL
firewire_firedtv	LD	14506	1367	1.7	331	0.9	0.4	0.2	3343	828
net_fddi_skfp	LD	30186	5041	11.2	6142	7.2	9.2	0.9	547	668
mtd_ubi	LD	39334	3633	7	MO	MO	4	0.9	908	>38
usb_core_main0	LD	52152	11084	2.7	4003	1.4	65	2	170	62
tty_synclinkmp	LD	19288	TO	TO	MO	MO	3.4	0.1	>4235	>1186
scsi_advansys	LD	21538	TO	TO	TO	TO	4	0.4	>3600	>3600
staging_vt6656	LD	25340	TO	TO	TO	TO	2	0.4	>7200	>7200
net_ppp	LD	15744	TO	TO	10530	0.15	924	0.3	>16	11.4
p10_100	CF	592	841	4.2	121	0.9	11	0.8	76	11
p16_140	CF	1783	MO	MO	MO	MO	11	3	>69	>24
p12_157	CF	4828	MO	MO	MO	MO	14	0.8	>71	>15
p13_153	CF	5816	MO	MO	MO	MO	54	2.7	>50	>26
p19_159	CF	9794	MO	MO	MO	MO	70	1.7	>15	>4
ddv_all	HM	6532	710	1.4	85	0.5	0.05	0.1	12772	1700

Experimental Evaluation

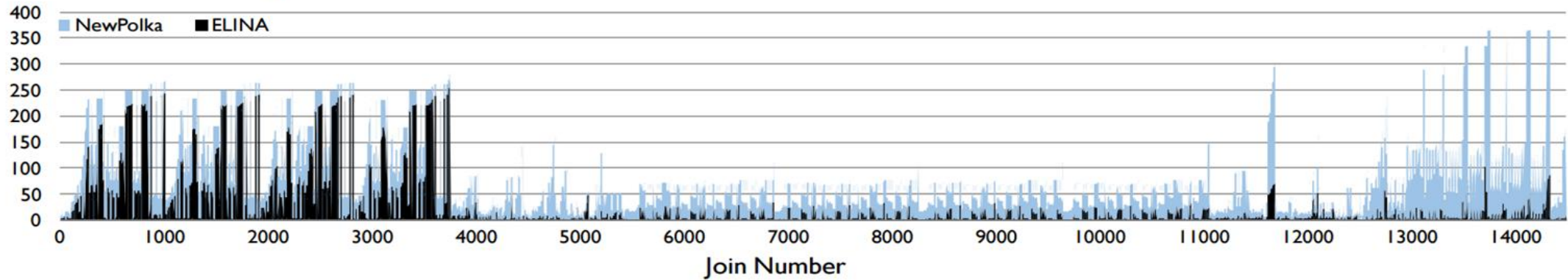
Benchmark	Category	LOC	NewPolka		PPL		ELINA		Speedup ELINA vs.	
			time(s)	memory(GB)	time(s)	memory(GB)	time(s)	memory(GB)	NewPolka	PPL
firewire_firedtv	LD	14506	1367	1.7	331	0.9	0.4	0.2	3343	828
net_fddi_skfp	LD	30186	5041	11.2	6142	7.2	9.2	0.9	547	668
mtd_ubi	LD	39334	3633	7	MO	MO	4	0.9	908	>38
usb_core_main0	LD	52152	11084	2.7	4003	1.4	65	2	170	62
tty_synclinkmp	LD	19288	TO	TO	MO	MO	3.4	0.1	>4235	>1186
scsi_advansys	LD	21538	TO	TO	TO	TO	4	0.4	>3600	>3600
staging_vt6656	LD	25340	TO	TO	TO	TO	2	0.4	>7200	>7200
net_ppp	LD	15744	TO	TO	10530	0.15	924	0.3	>16	11.4
p10_100	CF	592	841	4.2	121	0.9	11	0.8	76	11
p16_140	CF	1783	MO	MO	MO	MO	11	3	>69	>24
p12_157	CF	4828	MO	MO	MO	MO	14	0.8	>71	>15
p13_153	CF	5816	MO	MO	MO	MO	54	2.7	>50	>26
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Experimental Evaluation

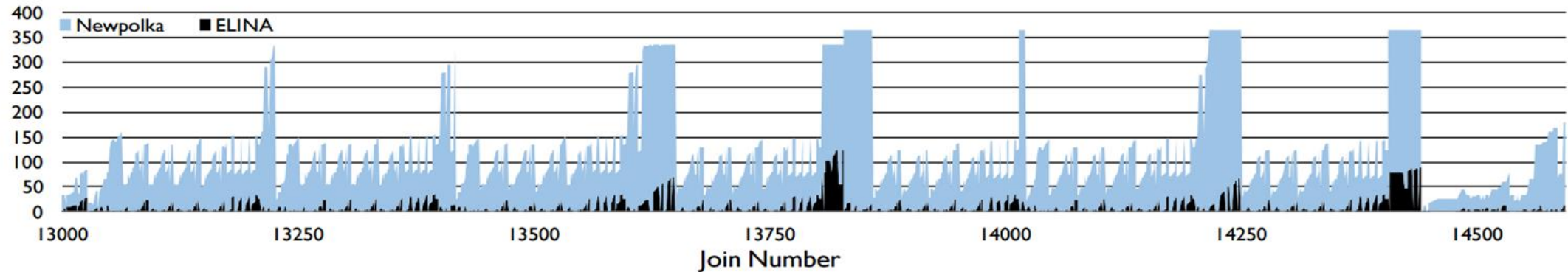
Benchmark	Category	LOC	NewPolka		PPL		ELINA		Speedup ELINA vs.	
			time(s)	memory(GB)	time(s)	memory(GB)	time(s)	memory(GB)	NewPolka	PPL
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Evaluation

Number of variables at join



Number of variables at join: zoom-in on 13000 onwards



$n_{ELINA} < n_{NewPolka}$, large speedup as conversion is exponential in n

Related Work

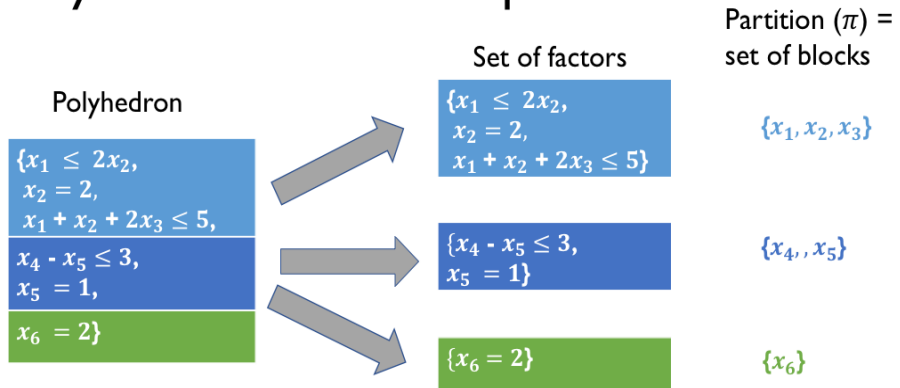
Related Work

- Variable Packing
 - Blanchet et al. [PLDI'03]
 - decomposition based on syntactic criteria
 - loses precision
- Matrix based decomposition
 - Halbwachs et al. [FMSSD'06]
 - does not work with generators
 - decomposition too coarse for join

Conclusion

Conclusion

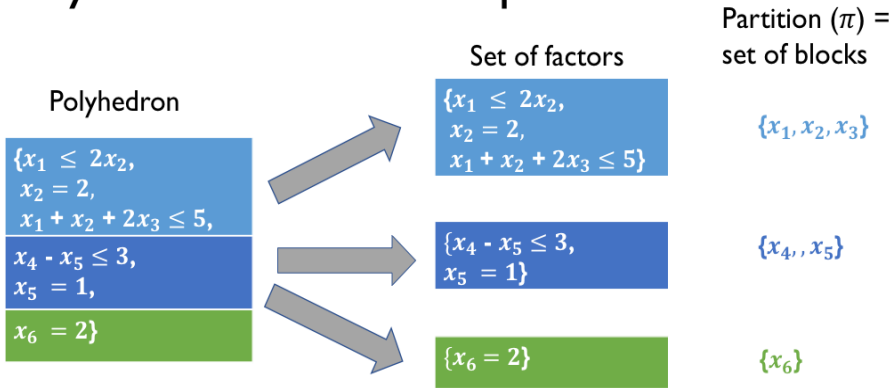
Key idea: online decomposition



Operators work on smaller Polyhedra: Complexity Reduction

Conclusion

Key idea: online decomposition

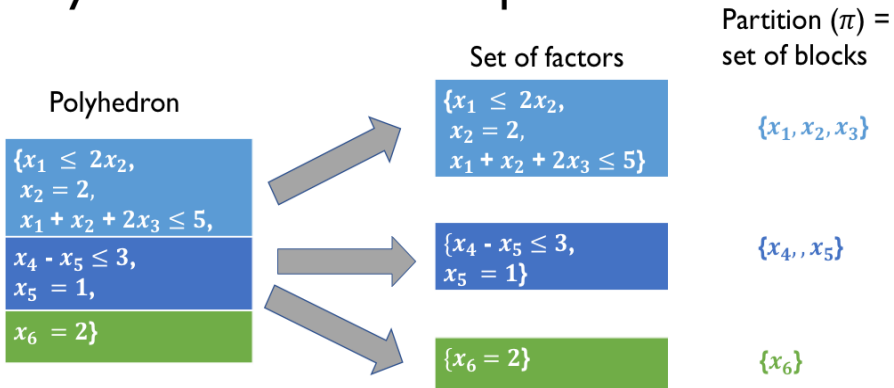


Operators work on smaller Polyhedra: Complexity Reduction

Operator	Both	Online decomposition
Join (\sqcup)	$O(n_g)$	$O(\sum_{i=1}^r n_i m_i g_i + n_{max} m_{max})$
Meet (\sqcap)	$O(nm)$	$O(\sum_{i=1}^r n_i m_i)$
Inclusion (\sqsubseteq)	$O(ngm)$	$O(\sum_{i=1}^r n_i m_i g_i)$
Assignment	$O(n_g)$	$O(n_{max} g_{max})$
Conditional	$O(n)$	$O(n_{max})$

Conclusion

Key idea: online decomposition



Operators work on smaller Polyhedra: Complexity Reduction

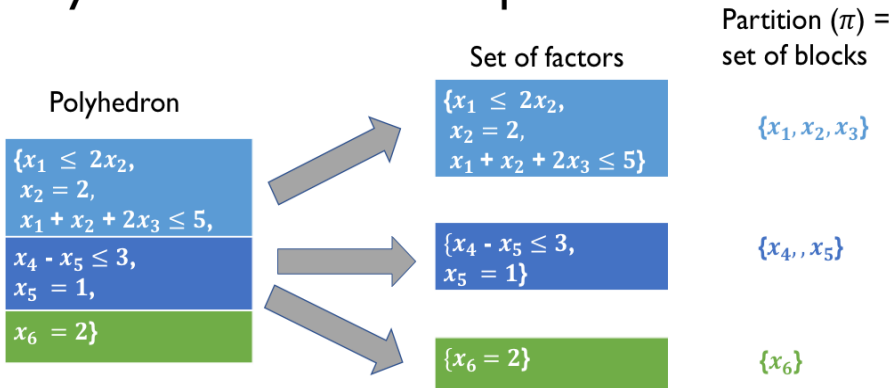
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ELNA

<http://elina.ethz.ch>

Conclusion

Key idea: online decomposition



Operators work on smaller Polyhedra: Complexity Reduction

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Join (\sqcup)	$O(ng)$	$O(\sum_{i=1}^r n_i m_i g_i + n_{max} m_{max})$
Meet (\sqcap)	$O(nm)$	$O(\sum_{i=1}^r n_i m_i)$
Inclusion (\sqsubseteq)	$O(ngm)$	$O(\sum_{i=1}^r n_i m_i g_i)$
Assignment	$O(ng)$	$O(n_{max} g_{max})$
Conditional	$O(n)$	$O(n_{max})$

ELINA

<http://elina.ethz.ch>

Driver	NewPolka	PPL	ELINA
➤ 500 var ➤ 39K LOC	OOM (> 12 GB)	OOM (> 12 GB)	4 sec 0.9 GB
➤ 650 var ➤ 25K LOC	TO (> 4 hr)	TO (> 4 hr)	2 sec 0.4 GB